

An Innovations Approach to Decoupling
of
Multibody Dynamics and Control

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Abstract

The paper solves the problem of hinged multibody dynamics using an extension of the innovations approach of linear filtering and prediction theory to the problem of mechanical system modeling and control. This approach has been used quite effectively to diagonalize the equations for filtering and prediction for linear state space systems. It has similar advantages in the study of dynamics and control of multibody systems. The innovations approach advanced here consists of expressing the equations of motion in terms of two closely related processes: (1) the innovations process e , a sequence of moments, obtained from the applied moments T by means of a spatially recursive Kalman filter that goes from the tip of the manipulator to its base; (2) a residual process, a sequence of velocities, obtained from the joint-angle velocities by means of an outward smoothing operations. The innovations e and the applied moments T are related by means of the relationships $e = (I - L)T$ and $T = (I + K)e$. The operation $(I - L)$ is a causal lower triangular matrix which is generated by a spatially recursive Kalman filter and the corresponding discrete-step Riccati equation. Hence, the innovations and the applied moments can be obtained from each other by means of a causal operation which is itself causally invertible. The residuals n and the joint-angle velocities \dot{q} are related by $n = (I - K^*)\dot{q}$ and $\dot{q} = (I - L^*)n$ in which $(I - L^*)$ is also an anticausal, upper-triangular, matrix. Hence, the residuals and the joint-angle velocities are related by means of an anticausal operation which is itself anticausally invertible. The use of the residuals process is of interest because it diagonalizes the composite multibody system kinetic energy. In other words, the kinetic energy $J(\dot{q}, q)$ in the system can be written as $J = 1/2 n^T D n$ in which D is a diagonal matrix. The Lagrangian equations of motion that result from this diagonal form for the kinetic energy are completely decoupled in the sense that the equation for the residual velocity at any given joint is independent from the similar equations at all of the remaining joints. The innovations process appears as a driving term in these equations. Use of the innovations, in place of the physically applied joint moments, decouples the equations even further. The equations of motion for joint k involves only the value of the innovations at the same joint. The final equations of motion are therefore diagonalized in the sense that the equation for any given joint is independent from the equations at the other

joints. The diagonal form of the equations of motion results in significant simplification of dynamic analysis, simulation, stability analysis, and control design. This simplicity is illustrated by arriving at a very simple decoupled control algorithm for robotic manipulator control.